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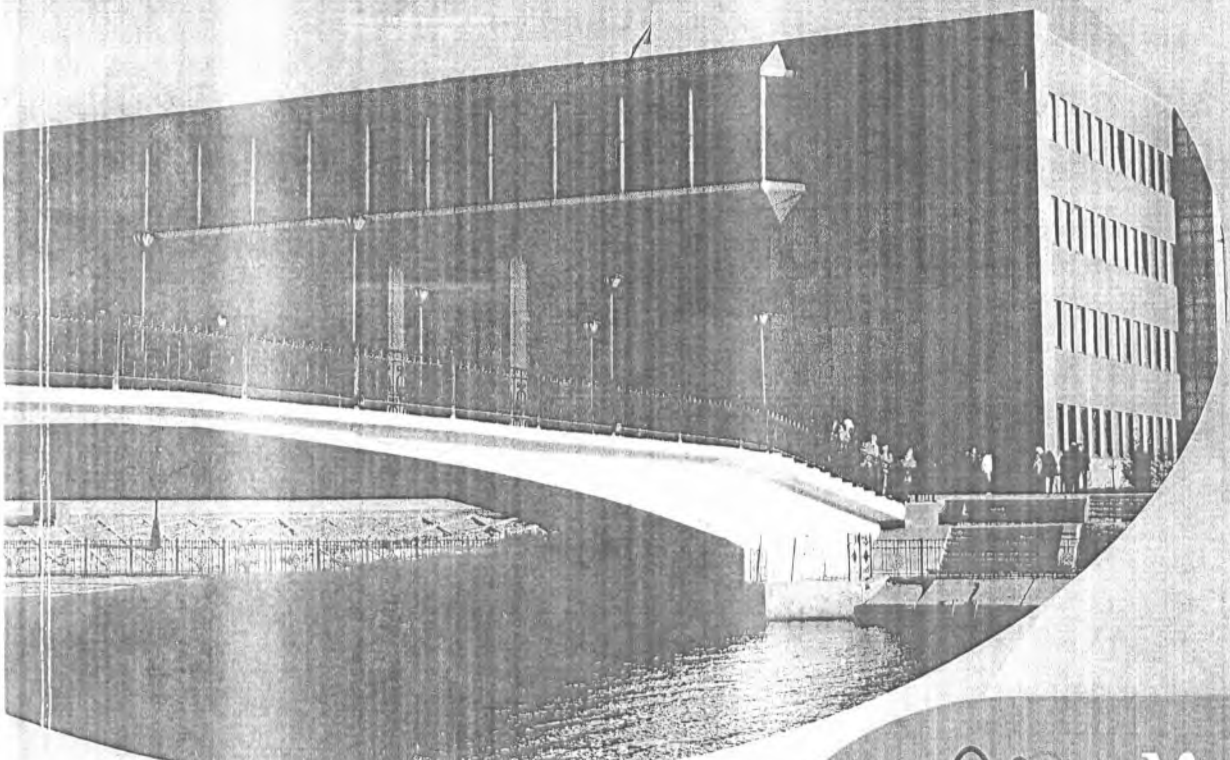

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МАЗМУНЫ СОДЕРЖАНИЕ

D.A. Iskenderova, A.M. Toktorbaev

Movement of a reacting gas mixture with contact discontinuity in Porous medium

(Kazakh Agro-Technical University, Astana c., Kazakhstan)
(Osh State University, Osh c., Kyrgyzstan)

The system of differential equations describing one-dimensional nonstationary flow of a reacting gas mixture in porous medium is considered [1]. Cauchy problem with discontinuity initial data corresponding to contact discontinuity is under study. A peculiar feature of flows with finite viscosity is unavailability of shock waves in them, i.e. there cannot be any other stronger discontinuity besides contact discontinuity. At the initial time the required functions tend to various constants at infinity. We shall consider mass Lagrangean coordinates [2].

I. Setting Problem and Basic Result. Let at the initial instant $t = 0$ a domain $-\infty < x < 0$ is occupied with gas with coefficients of viscosity, heat conductivity, diffusion, magnetic characteristics $\mu_1, \lambda_1, \chi_1, \nu_1$ and constitutive equation $p = r_1\rho\theta, \delta_{i1}-$ combination heat of i th component at standard conditions and a domain $0 < x < \infty$ occupied with gas with relevant characteristics $\mu_2, \lambda_2, \chi_2, \nu_2, \delta_{i2}$ and $p = r_2\rho\theta$.
Here $\mu_1, \lambda_1, \chi_1, \nu_1, \delta_{ij}, r_i(i, j = 1, 2)-$ are positive constants. Let us introduce notations:

$$\Omega_1 = \{x : -\infty < x < 0\}, \quad \Omega_2 = \{x : \infty < x < 0\}, \quad R = \Omega_1 \cup \Omega_2,$$

$$\Pi_{it} = \Omega_i \times (0, t), \quad \Gamma = \{x, t : x = 0, t \geq 0\},$$

$$v = \rho^{-1}, \quad \sigma = \mu\rho u_x - p, \quad p = r\rho\theta, \quad \delta = \delta_{i1} - \delta_{i2} \geq 0, \quad i = 1, 2,$$

where $x = 0-$ a line of contact discontinuity.
The environment behavior in the domain $-\infty < x < \infty$ is described as follows. Movement of each gas mixture outside the line of contact discontinuity is defined by equations:

$$\begin{aligned} &\frac{\partial v}{\partial t} - \frac{\partial u}{\partial x} = 0, \quad v = \frac{1}{p}, \\ &\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}(\frac{\chi}{v}\frac{\partial c}{\partial x}) - cg, \\ &\frac{\partial u}{\partial t} = \frac{\partial \sigma}{\partial x} - \beta(x)|u|^a u, \\ &\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}(\frac{\lambda}{v}\frac{\partial \theta}{\partial x}) + \frac{\partial}{\partial x}(\frac{\nu}{v}\theta\frac{\partial c}{\partial x}) + \sigma\frac{\partial u}{\partial x} + \delta cg. \end{aligned} \tag{1}$$

where $\beta(x)-$ coefficient of penetration is continuous positive bounded function and

$$\int_{-\infty}^{\infty} \beta(x)dx \leq C; \quad 0 \leq a \leq \frac{1}{2}.$$

Conditions of contact discontinuity at lines $x = 0$ are as follows:

$$[u] = [\theta] = [c] = [\sigma] = [\frac{\lambda}{v}\frac{\partial \theta}{\partial x}] = [\frac{\nu}{v}\theta\frac{\partial c}{\partial x}] = [\frac{\chi}{v}\frac{\partial c}{\partial x}] = 0, \quad (x = 0) \tag{2}$$

where: $[f] = f(+0, t) - f(-0, t)-$ a jump f.
At the initial instant $t = 0$ values of functions v, u, θ, c are supposed to be known:

$$u|_{t=0} = u_0(x), \theta|_{t=0} = \theta_0(x), c|_{t=0} = c_0(x), v|_{t=0} = v_0(x), |x| < \infty, \tag{3}$$

moreover, v_0, u_0, θ_0, c_0- are continuous at $x \neq 0$ and satisfy conditions (2) at x=0, $0 < m_0 \leq (v_0(x), \theta_0(x)) \leq M_0 < \infty, 0 < c_0(x) \leq 1$ and have finite limits at infinity:

$$\lim_{x \to -\infty} u_0(x) = u_0^1, \quad \lim_{x \to +\infty} u_0(x) = u_0^2, \quad u_0^1 < u_0^2,$$

$$\lim_{x\to-\infty} v_0(x) = v_0^1, \lim_{x\to+\infty} v_0(x) = v_0^2, \quad v_0^1 \neq v_0^2,$$
$$\lim_{x\to-\infty} \theta_0(x) = \theta_0^1, \lim_{x\to+\infty} \theta_0(x) = \theta_0^2, \quad \theta_0^1 \neq \theta_0^2. \tag{4}$$
$$\lim_{x\to-\infty} c_0(x) = c_0^1, \quad \lim_{x\to+\infty} c_0(x) = c_0^2, \quad c_0^1 \neq c_0^2,$$

Let us introduce auxiliary functions $\psi(x), f(x), \gamma(x), \varphi(x)$, possessing the following properties:

$$0 < C_1^{-1} < \psi(x) < C_1, \quad \lim_{|x|\to\infty} v_0(x)\psi(x) = 1, \psi'(x) \in W_2^1(R),$$
$$| f(x) |< C_2 < \infty, \lim_{x\to-\infty} f(x) = u_0^1, \quad \lim_{x\to+\infty} f(x) = u_0^2,$$
$$0 < f'(x) \leq C_3, \quad f'(x) \in W_2^1(R), \quad f'(x) \in L_1(R), \tag{5}$$
$$0 < C_4^{-1} < \varphi(x) < C_4, \quad \lim_{|x|\to\infty} \theta_0(x)\varphi(x) = 1, \quad \varphi'(x) \in W_2^1(R),$$
$$1 \leq \gamma(x) < C_5 < \infty, \quad \lim_{|x|\to\infty} c_0(x)\gamma(x) = 1, \quad \gamma'(x) \in W_2^1(R).$$
$$(\varphi'(x))^2 < \delta f'(x), \quad 0 < \delta < 1.$$

and

$$[\psi] = [\varphi] = [f] = [\eta] = 0 \quad (x = 0). \tag{6}$$

It is not difficult to check the existence of such functions.
THEOREM. *Let the initial data (3) satisfy conditions (4),*

$$(u_0 - f, v_0\psi - 1, \theta_0\varphi - 1, c_0\gamma - 1) \in W_1^2(\Omega_i)(i = 1, 2).$$

Function $g(p, c, \theta)$ is positive and continuous in any compact domain of its arguments, and besides, according to $(\varphi\theta)^{1/2}$, satisfies Lipschitz condition and $g(p, c, 1) = 0$.
Then there exists the unique generalized solution of the problem (1)-(3) "in general"according to time, at that:

$$(v\psi - 1) \in L_\infty\Big(0, T; W_1^2(\Omega_i)\Big),$$
$$\frac{\partial v}{\partial t} \in L_\infty\Big(0, T; L_2(\Omega_i)\Big), \left(\frac{\partial u}{\partial t}, \frac{\partial \theta}{\partial t}, \frac{\partial c}{\partial t,}\right) \in L_2(\Pi_{it}),$$
$$(u - f, \theta\varphi - 1, c\gamma - 1) \in L_\infty\Big(0, T; W_1^2(\Omega_i)\Big) \cap L_2\Big(0, T; W_2^2(\Omega_i)\Big), \quad (i = 1, 2),$$

$0 < c(x, t) \leq 1$, $\theta(x, t), v(x, t)-$ *strictly positive bounded functions,* $t \in [0, T], 0 < T < \infty$.
We shall prove a theorem by a method of a priori estimates. The existence of a local solution of problem (1)-(3) follows from [1-3]. Our aim is to find global a priori bounds, in which the constants C, C_i, K_i depend only on the data and the value of T , the length of the time interval.
2. A priori Estimates. Without loss of generality we can assume for simplicity that the physical parameters are equal to unity. It is evident from equations (1) and restrictions on the problem data that functions $v(x, t), and \theta(x, t)$ are not negative and

$$0 < c(x, t) \leq 1. \tag{7}$$

Let us derive a conservation law. We shall make substitutes, supposing that $\frac{\partial \xi}{\partial x} = \frac{1}{\varphi(x)\gamma(x)}$. Then a set of equations (1) will be as follows:

$$\frac{\partial v}{\partial t} - \frac{1}{\varphi\gamma}\frac{\partial u}{\partial \xi} = 0, \quad v = \frac{1}{p},$$

$$\frac{\partial c}{\partial t}=\frac{1}{\varphi\gamma}\frac{\partial}{\partial\xi}\Big(\frac{1}{\varphi\gamma v}\frac{\partial c}{\partial\xi}\Big)-cg,$$

$$\frac{\partial u}{\partial t}=\frac{1}{\varphi\gamma}\frac{\partial}{\partial\xi}\Big(\frac{1}{\varphi\gamma v}\frac{\partial u}{\partial\xi}\Big)-\frac{1}{\varphi\gamma}\frac{\partial p}{\partial\xi}-\beta(x)|u|^a u,\quad p=\frac{\theta}{v},\tag{8}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\varphi\gamma}\frac{\partial}{\partial\xi}\Big(\frac{1}{\varphi\gamma v}\frac{\partial\theta}{\partial\xi}\Big)+\frac{1}{\varphi\gamma}\frac{\partial}{\partial\xi}\Big(\frac{1}{\varphi\gamma v}\theta\frac{\partial c}{\partial\xi}\Big)-\frac{1}{\varphi\gamma}p\frac{\partial u}{\partial\xi}+\frac{1}{\varphi^2\gamma^2 v}\Big(\frac{\partial u}{\partial\xi}\Big)^2+cg.$$

Lemma 1. At fulfillment of the theorem conditions the following estimate is true

$$U(t)+\int_0^t W(\tau)\partial\tau\le E=const>0,\quad t\in[0,T]\tag{9}$$

where

$$U(t)=\int\Big\{\frac{1}{2}(u-f)^2+\frac{1}{2}(c\gamma-1)^2+(\varphi\theta-ln\varphi\theta-1)+(v\psi-lnv\psi-1)\Big\}\partial x,$$

$$W(t)=\int\Big\{\frac{\theta_x^2}{v\theta}+\frac{u_x^2}{v\theta}+\frac{c_x^2}{v}+\frac{\theta}{v}f'+g\varphi(c\gamma-1)^2+\beta(x)|u|^a(u-f)^2\Big\}\partial x.$$

The interval of integration with respect to x is from $-\infty$ to ∞.

Proving:

We multiply the first equation of the set (8) by $\gamma\Big(\psi-\frac{1}{v}\Big)$, the second equation - by $\gamma(c\gamma-1)$, the third equation - by $\varphi\gamma(u-f)$, the third - by $\gamma(\varphi-\frac{1}{\theta})$, add and integrate over R:

$$\frac{\partial}{\partial t}\int\Big\{\frac{1}{2}\varphi\gamma(u-f)^2+\frac{1}{2}(c\gamma-1)^2+\gamma(\varphi\theta-ln\varphi\theta-1)+\gamma(v\psi-lnv\psi-1)\Big\}\partial\xi,+$$

$$+\int\Big\{\frac{\theta_\xi^2}{v\theta\varphi^2\gamma}+\frac{u_\xi^2}{v\theta\varphi\gamma}+\frac{c_\xi^2}{v\varphi^2}+\frac{\theta}{v}f'+g(c\gamma-1)^2+\beta(\xi)|u|^a(u-f)^2\Big\}\partial\xi=$$

$$=\int\frac{\psi}{\varphi\gamma}u_\xi\partial\xi+\int\frac{1}{\varphi v\gamma}u_\xi(f'+\gamma-1)\partial\xi-\int\frac{\theta_\xi\varphi'}{v\theta\varphi^3\gamma}\partial\xi-\int\frac{c_\xi c\gamma'}{v\varphi^2\gamma}\partial\xi+\int\frac{c_\xi c\varphi'}{v\varphi^3}\partial\xi+$$

$$+\int\frac{c_\xi\theta_\xi}{v\theta\varphi^2\gamma}\partial\xi+\int\frac{c_\xi\varphi'}{v\varphi^3\gamma}\partial\xi-\int g(c\gamma-1)\partial\xi+$$

$$+\int cg\gamma\frac{\varphi\theta-1}{\theta}\partial\xi+\int\varphi\gamma\beta(\xi)|u|^a f(u-f)\partial\xi=\sum_{k=1}^{10}I_k.\tag{10}$$

Let us estimate each I_k, using integration by parts, Young, Cauchy, Holder inequalities, inclusions.

$$I_1=\int\frac{\psi}{\varphi\gamma}(u-f)\xi\partial\xi-\int\frac{\psi}{\varphi\gamma}f'\partial\xi\le\widetilde{N}_6\Big(\|\sqrt{\varphi\gamma}(u-f)\|^2+1\Big),$$

$$I_2=-\frac{\partial}{\partial t}\int(f'+\gamma-1)(v\psi-lnv\psi-1)\partial\xi+\int(f'+\gamma-1)\frac{\psi}{\varphi\gamma}u_\xi\partial\imath\xi\le$$

$$\le-\frac{\partial}{\partial t}\int(f'+\gamma-1)(v\psi-lnv\psi-1)\partial\xi+C_7(\|\sqrt{\varphi\gamma}(u-f)\|^2+1),$$

$$I_3=-\int\frac{\theta_\xi\varphi'\psi^{1/2}}{v^{1/2}\theta\varphi^3\gamma}\partial\xi+\int\frac{\theta_\xi\varphi'\psi^{1/2}((v\psi)^{1/2}-1)}{v^{1/2}\theta\varphi^3\gamma(v\psi)^{1/2}\sqrt{v\psi-lnv\psi-1}}\sqrt{v\psi-lnv\psi-1}\partial\xi.$$

Note that:

$$\frac{|(v\psi)^{1/2}-1|}{(v\psi)^{1/2}\sqrt{v\psi-\ln v\psi-1}}\le C_8,\quad \forall(x,t)\in\Pi.$$

Then

$$I_3\le\delta_1\int\frac{\theta_\xi^2}{v\theta^2\varphi^2\gamma}\partial\xi+C_{\delta 1}\Big(\int(v\psi-\ln v\psi-1)\partial\xi+1\Big).$$

Speculating in a similar way, it is possible to assess the remained integrals.

$$I_4\le\delta_2\int\frac{C_\xi^2}{v\varphi^2}\partial\xi+C_{\delta 2}\Big(\int(v\psi-\ln v\psi-1)\partial\xi+1\Big),$$

$$I_5\le\delta_3\int\frac{C_\xi^2}{v\varphi^2}\partial\xi+C_{\delta 3}\Big(\int(v\psi-\ln v\psi-1)\partial\xi+1\Big),$$

$$I_6\le\frac{1}{2}\int\frac{\theta_\xi^2}{v\theta^2\varphi^2\gamma}\partial\xi+\frac{1}{2}\int\frac{c_\xi^2}{v\varphi^2}\partial\xi,$$

$$I_7\le\delta_4\int\frac{c_\xi^2}{v\varphi^2}\partial\xi+C_{\delta 4}\Big(\int(v\psi-\ln v\psi-1)\partial\xi+1\Big).$$

I_8, I_9 are assessed taking into account Lipschitzness of function $g(\rho,c,\theta)$ according to $(\varphi\theta)^{1/2}$ and inequalities

$$\frac{|(\varphi\theta)^{1/2}-1|}{\sqrt{\varphi\theta-\ln\varphi\theta-1}}\le C_9,\quad\forall(x,t)\in\Pi. \tag{11}$$

$$I_8\le K_0\int\frac{|(\varphi\theta)^{1/2}-1|}{\sqrt{\varphi\theta-\ln\varphi\theta-1}}\sqrt{\varphi\theta-\ln\varphi\theta-1}\,|\,c\gamma-1\,|\,\partial\xi\le$$

$$\le C_{10}\Big[\int\gamma(\varphi\theta-ln\varphi\theta-1)\partial\xi+\frac{1}{2}\int(c\gamma-1)^2\partial\xi\Big].$$

Further, let us break down a number axis R into domains $\Omega_i(t)$ as follows:

$$\Omega_1(t)=\{x\in R:\varphi(x)\theta(x,t)\le 1\},\quad \Omega_2(t)=\{x\in R:\varphi(x)\theta(x,t)\le 1\},$$

Then

$$I_9=\int cg\gamma\frac{\varphi\theta-1}{\theta}\partial x=\int_{\Omega_1(t)}cg\gamma\frac{\varphi\theta-1}{\theta})\partial x+\int_{\Omega_2(t)}cg\gamma\frac{\varphi\theta-1}{\theta}\partial x\le\int_{\Omega_2(t)}cg\gamma\frac{\varphi\theta-1}{\theta}\partial x$$

pursuant to positivity of functions $g(\rho,c,\theta)$ and $\tilde{n}(x,t)$.
Note that in $\Omega_2(t)$ the inequality is realized:

$$\frac{\Big((\varphi\theta)^{1/2}-1\Big)(\varphi\theta-1)}{\varphi\theta(\varphi\theta-\ln\varphi\theta-1)}\le C_{11},\quad\forall(x,t)\in\Pi.$$

Returning to I_9 , we have

$$I_9\le K_0C_4C_5\int_{\Omega_2(t)}\frac{\Big((\varphi\theta)^{1/2}-1\Big)(\varphi\theta-1)}{\varphi\theta(\varphi\theta-\ln\varphi\theta-1)}(\varphi\theta-\ln\varphi\theta-1)\partial\xi\le C_{12}\int(\varphi\theta-\ln\varphi\theta-1).$$

And

$$I_{10} \le \delta_5 \int \beta(\xi) \mid u \mid^a (u-f)^2 \varphi\gamma\partial\xi + C_{\delta 5}\left[\int \beta(\xi)|u-f|^a f^2\varphi\gamma\partial\xi + \int \beta(\xi)|f|^a f^2 \varphi\gamma\partial\xi\right] \le$$

$$\le \delta_5 \int \beta(\xi) \mid u \mid^a (u-f)^2\varphi\gamma\partial\xi + C_{13}\left[\left(\int \varphi(u-f)^2\partial\xi\right)^{\frac{a}{2}}\left(\int (\beta(\xi)f^2)^{\frac{2}{2-a}}\partial\xi\right)^{\frac{2-a}{2}} + 1\right] \le$$

$$\le \delta_5 \int \beta(\xi) \mid u \mid^a (u-f)^2\varphi\gamma\partial\xi + C_{14}\Big(l \parallel \sqrt{\varphi\gamma} \parallel^2 + 1\Big).$$

Select $\sum_{i=1}^{5} \delta_i < 1$. By integrating the inequality obtained from (10) with respect to time and using Gronwall's lemma [4] we get (9) after returning to the old independent variable x.
Lemma is proved.

3. Auxiliary Relation between Desired Functions. Following [4], let us divide the number axis R and, accordingly, the strip Π into final segments and rectangles:

$$R = \bigcup_{N=-\infty}^{\infty} \overline{\Omega}_N, \Pi = \bigcup_{N=-\infty}^{\infty} \overline{Q}_N,$$

$$\Omega_N = \{x \mid N < x < N = 1\}, \quad Q_N = \Omega_N \times (0,T), N = 0, \pm 1, \pm 2, ...$$

Let us take at random one of such rectangles. Since in (9) functions $(v\psi - \ln v\psi - 1), (\varphi\theta - \ln\varphi\varphi - 1)$ are negative at $v > 0, \theta > 0$, then

$$U_N(t) + \int_0^t W_N(\tau)\partial\tau \le E$$

where integrals in the definition U_N and W_N are taken according to Ω_N.
According to [4] positive constants $n(E), M(E)$, exist here, which do not depend on N :

$$\frac{n(E)}{\widetilde{N}_1} \le \int_N^{N+1} v(x,t)\partial x \le M(E)\widetilde{N}_1, \frac{n(E)}{\widetilde{N}_4} \le \int_N^{N+1} \theta(x,t)\partial x \le M(E)\widetilde{N}_4, \quad \forall t \in [0,T]. \qquad (12)$$

One auxiliary relation between desired functions in each of rectangles $\overline{Q}_N$. is derived from the first and third equations of the set (1) as in [4].

$$v(x,t) = I^{-1}(t)B^{-1}(x,t)S(x,t)\Big[v_0(x) + \int_0^t \theta(x,\tau)I(\tau)B(x,\tau)S^{-1}(x,\tau)\partial\tau\Big], \qquad (13)$$

where:

$$I(t) = I_N(t) = \frac{v_0(a(t))}{v(a(t),t)} exp\left\{\int_0^t \theta(a(t),\tau)\partial\tau\right\},$$

$$B(x,t) = B_N(x,t) = exp\left\{\int_{a(t)}^x (u_0(\xi) - u(\xi,t))\partial\xi\right\}.$$

$$S(x,t) = exp\left\{\int_0^t \int_{a(t)}^x \beta(\xi) \mid u \mid^a u(\xi,\tau))\partial\xi\partial\tau\right\}.$$

True estimates are as follows:

$$0 < K_1^{-1} \le B(x,t) \le K_1, 0 < K_2^{-1} \le I(t) \le K_2, 0 < K_3^{-1} \le S(x,t) \le K_3, \qquad (14)$$

$$\forall x \in \overline{\Omega}_N, \quad \forall t \in [0,T]$$

The proof. To prove the bounds $S(x,t)$.

$$\left| \int_0^t \int_{a(t)}^x \beta(\xi) \mid u \mid^a u(\xi,\tau))\partial\xi\partial\tau\} \right| \le \int_0^t \int_N^{N+1} \beta(x) \mid u - f \mid^{a+1} \partial x \partial\tau +$$

$$+ \int_0^t \int_N^{N+1} \beta(x) \mid f \mid^{a+1} \partial x\partial\tau \le\le \int_0^t \left(\int_N^{N+1} (u-f)^2 \partial x \right)^{\frac{1+a}{2}} \left(\int_N^{N+1} \beta^{\frac{2}{1-a}}(x) \partial x \right)^{\frac{1-a}{2}} + C \le C_{15}$$

The bounds $B(x,t), I(t)$, independent of N follows from estimates (9) and representation (13) as in [4].

*4. **Estimates for Density (Specific Volume) and Temperature.***

$$M_h(t) = \max_{|x|<\infty} h(x,t), \quad m_h(t) = \min_{|x|<\infty} h(x,t).$$

Lemma 2. At fulfillment of the theorem conditions the following estimates are true:

$$m_v(t) \le K_4, \quad m_\theta(t) \le K_5, \quad \forall t \in [0,T].$$

Proof. Strict positivity of specific volume follows from representation (13) taking into account theorem conditions and (14), strict positivity of temperature follows from thermal-conductivity equation of the set (1).

Lemma 3At fulfillment of the theorem conditions the following estimate is true:

$$M_v(t) \le K_6, \quad \forall t \in [0,T].$$

Proof. The following estimates take place [4]:

$$M_\theta(t) \le C_\epsilon A(t) M_v(t) + C, \quad where \quad A(t) = \int \frac{\theta_x^2}{v\theta^2} \partial x \tag{15}$$

Applying Gronwall's lemma to (13), taking into account estimates (14), (15), we shall receive boundedness of specific volume from above. From lemma 3, taking into account (9) и (15), the following estimate arises

$$\int_0^t M_\theta(t)\partial t \le K_3, \quad \forall t \in [0,T]. \tag{16}$$

5. Estimates for Derivatives from Desired Functions. Let us multiply the second equation of the set (1) by $\frac{\partial}{\partial x}(\frac{1}{v})c_x$ and integrate over R :

$$\frac{1}{2}\frac{\partial}{\partial t} \int \frac{1}{v} c_x^2 \partial x + \int \left(\frac{1}{v} c_x\right)_x^2 \partial x = -\frac{1}{2} \int \frac{1}{v^2} c_x^2 u_x \partial x + \int cg \left(\frac{1}{v} c_x\right)_x \partial x = I_1 + I_2 \tag{17}$$

Using integration by parts, Young, Cauchy inequalities, embeddings, (9), Lipschitzness of function g according to $(\varphi\theta)^{1/2}$, we shall estimate $I_k, k = 1,2$.

$$I_1 = \int \left(\frac{1}{v} c_x\right) \left(\frac{1}{v} c_x\right)_x (u - f) \partial x - \frac{1}{2} \int \frac{1}{v^2} c_x^2 f' \partial x \le$$

$$\leq \max_{x\in R}\left|\frac{1}{v}c_x\right|\Big(\int(\frac{1}{v}c_x)_x^2\partial x\Big)^{1/2}+\frac{C_3}{2K_4}\int\frac{1}{v}c_x^2\partial x.$$

As

$$\max_{x\in R}\Big(\frac{1}{v}c_x\Big)^2\leq 2\int\left|\frac{1}{v}c_x\Big(\frac{1}{v}c_x\Big)_x\right|\partial x\leq\frac{2}{K_4^{1/2}}\Big(\int\frac{1}{v}c_x^2\partial x\Big)^{1/2}\Big(\int(\frac{1}{v}c_x)_x^2\partial x\Big)^{1/2},$$

then

$$I_1\leq\delta_1\int\Big(\frac{1}{v}c_x\Big)_x^2\partial x+c_{\delta 1}\int\frac{1}{v}c_x^2\partial x.$$

Let us estimate I_2, taking into account (9), (11).

$$I_2\leq K_0C_9\Big(\int(\varphi\theta-ln\varphi\theta-1)\partial x\Big)^{1/2}\left(\int\Big(\frac{1}{v}c_x\Big)_x^2\partial x\right)^{1/2}\leq\delta_2\int\Big(\frac{1}{v}c_x\Big)_x^2\partial x+C_{\delta 2}.$$

Select $\delta_i, i=1,2$ rather small. By integrating the inequality obtained from (17) with respect to time and using Gronwall's lemma, we shall find:

$$\int\frac{1}{v}c_x^2\partial x+\int_0^t\int\Big(\frac{1}{v}c_x\Big)_x^2\partial x\partial\tau\leq K_8,\quad\forall t\in[0,T].\qquad(18)$$

Let us multiply the third equation of the set (1) by $(u-f)$ and integrate over R :

$$\frac{1}{2}\frac{\partial}{\partial t}\int(u-f)^2\partial x+\int\left\{\frac{u_x^2}{v}+\frac{\theta}{v}f'\right\}\partial x+\int\beta(x)|u|^a(u-f)^2\partial x=$$

$$=\int\frac{1}{v}u_xf'\partial x+\int\frac{\theta}{v}u_x\partial x+\int\beta(x)|u|^a(u-f)^2\partial x=J_1+J_2+J_3.\qquad(19)$$

We shall estimate each $J_k, k=1,2,3$, according to Cauchy inequality, using (5), (9), (11).

$$J_1\leq-\frac{\partial}{\partial t}\int f'(v\psi-lnv\psi-1)\partial\xi+C,$$

$$J_2=\int\frac{\varphi\theta-1}{\varphi v}u_x\partial x+\int\frac{1}{\varphi v}\partial x\leq$$

$$\leq\frac{1}{C_4}\int\frac{\left|(\varphi\theta)^{1/2}-1\right|\Big((\varphi\theta)^{1/2}+1\Big)}{\sqrt{\varphi\theta-ln\varphi\theta-1}}\sqrt{\varphi\theta-ln\varphi\theta-1}\frac{|u_x|}{v}\partial x-\frac{\partial}{\partial t}\int\frac{1}{\varphi}(v\psi-lnv\psi-1)\partial\xi\leq$$

$$\leq-\frac{\partial}{\partial t}\int\frac{1}{\varphi}(v\psi-lnv\psi-1)\partial\xi+\delta\int\frac{1}{v}u_x^2\partial x+C(1+M_\theta(t)).$$

$$J_3\leq\epsilon\int\beta(\xi)|u|^a(u-f)^2\partial\xi+C_{16}.$$

From (19), after integrating over t , taking into account (9), (16), we derive:

$$\int_0^t\|u_x\|^2\partial\tau\leq K_9,\forall t\in[0,T].$$

Arguing analogously it is possible to obtain all estimates required for proving the existence of the generalized solution as in [5]. The uniqueness is proved by homogeneous equation composed regarding the difference of two possible solutions similarly to [2, 5].

The theorem is fully proved.

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Искендерова Д.А., Тоқтарбаев А.М.

Кеуектілік ортада үзілісті байланыспен өзгеретін газ қоспаларының қозғалысы

Кеуектілік ортада өзгеретін газ қоспаларының біркелкі стационарлық емес қозғалысын сипаттайтын дифференциалдық теңдеулер жүйесі зерттелуде Коши есептері үзілісті байланысқа сәйкес бастапқы үзілісті мәліметтермен оқытылуда. Уақыттың бастапқы кезеңінде барлық ізденіс функциялары әр түрлі тұрақты шексіздікке ұмтылады.

D.A. Iskenderova, A.M. Toktorbaev

Movement of a reacting gas mixture with contact discontinuity in porous medium

The system of differential equations describing one-dimensional nonstationary flow of a reacting gas mixture in porous medium is considered. Cauchy problem with discontinuity initial data corresponding to contact discontinuity is under study. At the initial time the required functions tend to various constants at infinity. We shall consider mass Lagrangean coordinates.

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