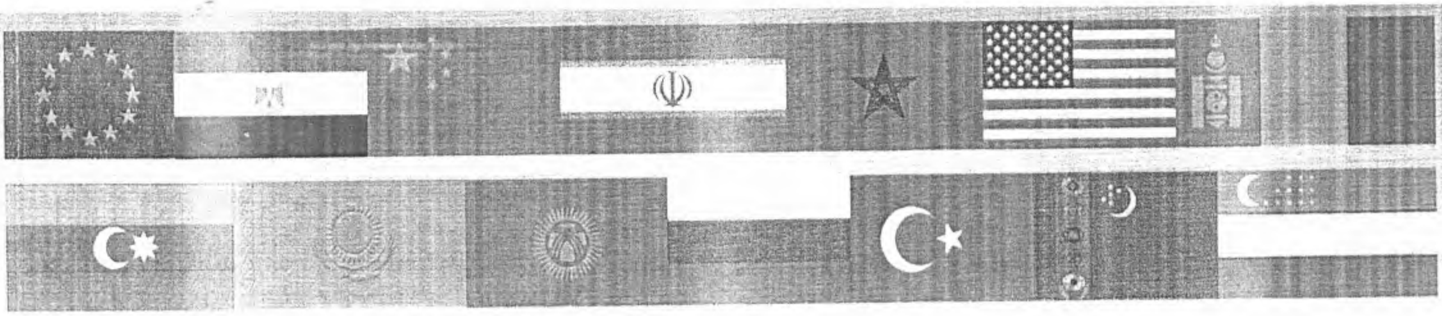

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MOVEMENT OF REACTING GAS MIXTURE WITH CONTACT DISCONTINUITY

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I. Setting Problem and Basic Result. Cauchy problem with discontinuity initial data corresponding to contact discontinuity is under study. A peculiar feature of flows with finite viscosity is unavailability of shock waves in them, i.e. there cannot be any other stronger discontinuity besides contact discontinuity. We shall consider mass Lagrangean coordinates.

Let at the initial instant $t=0$ a domain $-\infty<x<0$ is occupied with gas with coefficients of viscosity, heat conductivity, diffusion, magnetic characteristics $\mu_1, \lambda_1, \chi_1, \nu_1$ and constitutive equation $p=r_1\rho\,\theta$, δ_{i1} - combination heat of $i\,th$ component at standard conditions and a domain $0<x<\infty$ is occupied with gas with relevant characteristics $\mu_2, \lambda_2, \chi_2, \nu_2, \delta_{i2}$ and $p=r_2\rho\,\theta$. Here $\mu_i, \lambda_i, \chi_i, \nu_i, \delta_{ij}, r_i\ (i,j=1,2)$ - are positive constants. Let us introduce notations:

$$\Omega_1=\{x:-\infty<x<0\},\quad \Omega_2=\{x:0<x<\infty\},\quad R=\Omega_1\cup\Omega_2,$$
$$\Pi_{it}=\Omega_i\times(0,t),\quad \Gamma=\{x,t:x=0,t\ge 0\},$$
$$\mathrm{v}=\rho^{-1},\ \sigma=\mu\rho\,u_x-p,\quad p=r\rho\,\theta,\quad \delta=\delta_{i1}-\delta_{i2}\ge 0,\quad i=1,2,$$

where $x=0$ - a line of contact discontinuity.

The environment behavior in the domain $-\infty<x<\infty$ is described as follows. Movement of each gas mixture outside the line of contact discontinuity is defined by equations:

$$\frac{\partial \mathrm{v}}{\partial t}-\frac{\partial u}{\partial x}=0,\quad \mathrm{v}=\frac{1}{\rho},$$
$$\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(\frac{\chi}{\mathrm{v}}\frac{\partial c}{\partial x}\right)-c\,g,$$
(1)
$$\frac{\partial u}{\partial t}=\frac{\partial \sigma}{\partial x},$$
$$\frac{\partial \theta}{\partial t}=\frac{\partial}{\partial x}\left(\frac{\lambda}{\mathrm{v}}\frac{\partial \theta}{\partial x}\right)+\frac{\partial}{\partial x}\left(\frac{\nu}{\mathrm{v}}\theta\frac{\partial c}{\partial x}\right)+\sigma\frac{\partial u}{\partial x}+\delta\,c\,g,$$

Conditions of contact discontinuity at lines $x=0$ are as follows:

$$[u]=[\theta]=[c]=[\sigma]=\left[\frac{\lambda}{\mathrm{v}}\frac{\partial \theta}{\partial x}\right]=\left[\frac{\nu}{\mathrm{v}}\theta\frac{\partial c}{\partial x}\right]=\left[\frac{\chi}{\mathrm{v}}\frac{\partial c}{\partial x}\right]=0,\quad (x=0) \tag{2}$$

where: $[f]=f(+0,t)-f(-0,t)$ - a jump f.

At the initial instant $t=0$ values of functions v,u,θ,c are supposed to be known:

$$u|_{t=0}=u_0(x),\quad \theta|_{t=0}=\theta_0(x),\quad c|_{t=0}=c_0(x),\quad \mathrm{v}|_{t=0}=\mathrm{v}_0(x), \tag{3}$$

moreover, $(\mathrm{v}_0, u_0, \theta_0, c_0)$ - are smooth at $x \neq 0$ and satisfy conditions (2) at $x = 0$, $0 < m_0 \le (\mathrm{v}_0(x), \theta_0(x)) \le M_0 < \infty$, $0 < c_0(x) \le 1$ and have finite bounds at infinity:

$$\lim_{x\to-\infty} u_0(x) = u_0^1, \quad \lim_{x\to+\infty} u_0(x) = u_0^2, \quad u_0^1 < u_0^2,$$
$$\lim_{x\to-\infty} \mathrm{v}_0(x) = \mathrm{v}_0^1, \quad \lim_{x\to+\infty} \mathrm{v}_0(x) = \mathrm{v}_0^2, \quad \mathrm{v}_0^1 \neq \mathrm{v}_0^2,$$
$$\lim_{x\to-\infty} \theta_0(x) = \theta_0^1, \quad \lim_{x\to+\infty} \theta_0(x) = \theta_0^2, \quad \theta_0^1 \neq \theta_0^2, \tag{4}$$
$$\lim_{x\to-\infty} c_0(x) = c_0^1, \quad \lim_{x\to+\infty} c_0(x) = c_0^2, \quad c_0^1 \neq c_0^2.$$

Let us introduce auxiliary functions $\psi(x)$, $f(x)$, $\gamma(x)$, $\varphi(x)$, possessing the following features:

$$0 < C_1^{-1} < \psi(x) < C_1, \quad \lim_{|x|\to\infty} \mathrm{v}_0(x)\psi(x) = 1, \quad \psi'(x) \in W_2^1(R),$$
$$|f(x)| < C_2 < \infty, \quad \lim_{x\to-\infty} f(x) = u_0^1, \quad \lim_{x\to+\infty} f(x) = u_0^2,$$
$$0 < f'(x) \le C_3, \; f'(x) \in W_2^1(R), \; f'(x) \in L_1(R), \tag{5}$$
$$0 < C_4^{-1} < \varphi(x) < C_4, \quad \lim_{|x|\to\infty} \theta_0(x)\varphi(x) = 1, \quad \varphi'(x) \in W_2^1(R),$$
$$1 \le \gamma(x) < C_5 < \infty, \quad \lim_{|x|\to\infty} c_0(x)\gamma(x) = 1, \quad \gamma'(x) \in W_2^1(R).$$

$$(\varphi'(x))^2 < \delta f'(x), \quad 0 < \delta < 1.$$

and $[\psi] = [\varphi] = [f] = [\eta] = 0 \quad (x = 0)$.

It is not difficult to check the existence of such functions.

Theorem. *Let the initial data (3) satisfy conditions (4),*

$$(u_0 - f, \; \mathrm{v}_0\psi - 1, \; \theta_0\varphi - 1, \; c_0\gamma - 1) \in W_2^1(\Omega_i) \quad (i = 1,2).$$

Function $g(\rho, c, \theta)$ is positive and continuous in any compact domain of its arguments, and besides, according to $(\varphi\theta)^{1/2}$ satisfies Lipschitz condition and $g(\rho, c, 1) = 0$.

Then there exists the only generalized solution of the problem (1)-(3) "in general" according to time, at that:

$$(\mathrm{v}\psi - 1) \in L_\infty\left(0, T; W_2^1(\Omega_i)\right),$$
$$\frac{\partial \mathrm{v}}{\partial t} \in L_\infty(0, T; L_2(\Omega_i)), \; \left(\frac{\partial u}{\partial t}, \frac{\partial \theta}{\partial t}, \frac{\partial c}{\partial t}, \right) \in L_2(\Pi_{it}),$$
$$(u - f, \varphi\theta - 1, \; c\gamma - 1) \in L_\infty\left(0, T; W_2^1(\Omega_i)\right) \cap L_2\left(0, T; W_2^2(\Omega_i)\right), \; (i = 1,2),$$

$0 < c(x,t) \le 1$, $\theta(x,t)$, $\mathrm{v}(x,t)$ - *strictly positive bounded functions*, $t \in [0, T]$.

We shall prove a theorem by a method of a priori estimates.

2. A priori Estimates. It is evident from equations (1) and restrictions on the problem data that functions $\mathrm{v}(x,t)$ and $\theta(x,t)$ are not negative and

$$0 < c(x,t) \le 1. \tag{7}$$

Let us derive a conservation law. We shall make substitutes, supposing that $\dfrac{\partial \xi}{\partial x} = \dfrac{1}{\varphi(x)\gamma(x)}$. Then a set of equations (1) will be as follows:

$$\frac{\partial \mathrm{v}}{\partial t}-\frac{1}{\varphi\gamma}\frac{\partial u}{\partial \xi}=0,\quad \mathrm{v}=\frac{1}{\rho},$$
$$\frac{\partial c}{\partial t}=\frac{1}{\varphi\gamma}\frac{\partial}{\partial \xi}\left(\frac{1}{\varphi\gamma\mathrm{v}}\frac{\partial c}{\partial \xi}\right)-cg,$$
$$\frac{\partial u}{\partial t}=\frac{1}{\varphi\gamma}\frac{\partial}{\partial \xi}\left(\frac{1}{\varphi\gamma\mathrm{v}}\frac{\partial u}{\partial \xi}\right)-\frac{1}{\varphi\gamma}\frac{\partial p}{\partial \xi},\quad p=\frac{\theta}{\mathrm{v}},\tag{8}$$
$$\frac{\partial \theta}{\partial t}=\frac{1}{\varphi\gamma}\frac{\partial}{\partial \xi}\left(\frac{1}{\varphi\gamma\mathrm{v}}\frac{\partial \theta}{\partial \xi}\right)+\frac{1}{\varphi\gamma}\frac{\partial}{\partial \xi}\left(\frac{1}{\varphi\gamma\mathrm{v}}\theta\frac{\partial c}{\partial \xi}\right)-\frac{1}{\varphi\gamma}p\frac{\partial u}{\partial \xi}+\frac{1}{\varphi^2\gamma^2\mathrm{v}}\left(\frac{\partial u}{\partial \xi}\right)^2+cg.$$

LEMMA 1. At fulfillment of the theorem conditions the following estimate is valid

$$U(t)+\int_0^t W(\tau)\,d\tau \le E=const>0,\quad t\in[0,T]\tag{9}$$

where: $U(t)=\int\left\{\frac{1}{2}(u-f)^2+\frac{1}{2}(c\gamma-1)^2+(\varphi\theta-\ln\varphi\theta-1)+(\mathrm{v}\psi-\ln \mathrm{v}\psi-1)\right\}dx,$

$$W(t)=\int\left\{\frac{\theta_x^2}{\mathrm{v}\theta^2}+\frac{u_x^2}{\mathrm{v}\theta}+\frac{c_x^2}{\mathrm{v}}+\frac{\theta}{\mathrm{v}}f'+g\varphi(c\gamma-1)^2\right\}dx.$$

Proving. We multiply the first equation of the set (8) by $\gamma\left(\psi-\frac{1}{\mathrm{v}}\right)$, the second equation - by $\gamma(c\gamma-1)$, the third equation - by $\varphi\gamma(u-f)$, the third - by $\gamma\left(\varphi-\frac{1}{\theta}\right)$, add and integrate over R:

$$\frac{d}{dt}\int\left\{\frac{1}{2}\varphi\gamma(u-f)^2+\frac{1}{2}(c\gamma-1)^2+\gamma(\varphi\theta-\ln\varphi\theta-1)+\gamma(\mathrm{v}\psi-\ln \mathrm{v}\psi-1)\right\}d\xi+$$
$$+\int\left\{\frac{\theta_\xi^2}{\mathrm{v}\theta^2\varphi^2\gamma}+\frac{u_\xi^2}{\mathrm{v}\theta\,\varphi\gamma}+\frac{c_\xi^2}{\mathrm{v}\,\varphi^2\gamma}+\frac{\theta}{\mathrm{v}}f'+g(c\gamma-1)^2\right\}d\xi=\tag{10}$$
$$=\int\frac{\psi}{\varphi\gamma}u_\xi d\xi+\int\frac{1}{\varphi\mathrm{v}\gamma}u_\xi(f'+\gamma-1)d\xi-\int\frac{\theta_\xi\varphi'}{\mathrm{v}\,\theta\,\varphi^3\gamma}d\xi-\int\frac{c_\xi c\gamma'}{\mathrm{v}\,\varphi^2\gamma}d\xi+\int\frac{c_\xi c\varphi'}{\mathrm{v}\,\varphi^3}d\xi+$$
$$+\int\frac{c_\xi\theta_\xi}{\mathrm{v}\,\theta\varphi^2\gamma}d\xi+\int\frac{c_\xi\varphi'}{\mathrm{v}\,\varphi^3\gamma}d\xi-\int g(c\gamma-1)d\xi+\int cg\gamma\frac{\varphi\theta-1}{\theta}d\xi=\sum_{k=1}^{9}I_k$$

Let us assess each I_k, using integration by parts, Young, Cauchy, Holder inequalities, inclusions.

$$I_1=\int\frac{\psi}{\varphi\gamma}(u-f)_\xi\,d\xi+\int\frac{\psi}{\varphi\gamma}f'\,d\xi\le C_6\left(\left\|\sqrt{\varphi\gamma}(u-f)\right\|^2+1\right),$$
$$I_2=\int(f'+\gamma-1)\frac{\partial\ln \mathrm{v}\psi}{\partial t}d\xi=-\int(f'+\gamma-1)\frac{\partial}{\partial t}(\mathrm{v}\psi-\ln \mathrm{v}\psi-1)d\xi+$$

$$+\int (f'+\gamma-1)\frac{\partial\, \mathrm{v}\psi}{\partial t}d\xi=-\frac{d}{dt}\int (f'+\gamma-1)(\mathrm{v}\psi-\ln \mathrm{v}\psi-1)\,d\xi+\int (f'+\gamma-1)\frac{\psi^{\mathrm{v}}}{\varphi\gamma}u_{\xi}d\xi\le$$

$$\le-\frac{d}{dt}\int (f'+\gamma-1)(\mathrm{v}\psi-\ln \mathrm{v}\psi-1)\,d\xi+C_7\left(\left\|\sqrt{\varphi\gamma}(u-f)\right\|^2+1\right),$$

$$I_3=\int\frac{\theta_\xi\varphi'\psi^{1/2}}{\mathrm{v}^{1/2}\,\theta\varphi^3\gamma}d\xi-\int\frac{\theta_\xi\varphi'\psi^{1/2}\left((\mathrm{v}\psi)^{1/2}-1\right)}{\mathrm{v}^{1/2}\,\theta\varphi^3\gamma(\mathrm{v}\psi)^{1/2}\sqrt{\mathrm{v}\psi-\ln \mathrm{v}\psi-1}}\sqrt{\mathrm{v}\psi-\ln \mathrm{v}\psi-1}d\xi\ .$$

Note that:

$$\frac{\left|(\mathrm{v}\psi)^{1/2}-1\right|}{(\mathrm{v}\psi)^{1/2}\sqrt{\mathrm{v}\psi-\ln \mathrm{v}\psi-1}}\le C_8,\quad \forall (x,t)\in\Pi\,.$$

Then

$$I_3\le\left(\int\frac{\theta_\xi^2}{\mathrm{v}\,\theta^2\varphi^2\gamma}d\xi\right)^{1/2}\left(\int\frac{\varphi'^2\psi}{\varphi^4\gamma}d\xi\right)^{1/2}+$$

$$+C_8\left(\int\frac{\theta_\xi^2}{\mathrm{v}\,\theta^2\varphi^2\gamma}d\xi\right)^{1/2}\left(\int\frac{\varphi'^2\psi}{\varphi^4\gamma}(\mathrm{v}\psi-\ln \mathrm{v}\psi-1)d\xi\right)^{1/2}\le$$

$$\le\delta_1\int\frac{\theta_\xi^2}{\mathrm{v}\,\theta^2\varphi^2\gamma}d\xi+C_{\delta 1}\left(\int(\mathrm{v}\psi-\ln \mathrm{v}\psi-1)d\xi+1\right).$$

Speculating in a similar way, it is possible to assess the remained integrals.

$$I_4\le\delta_2\int\frac{c_\xi^2}{\mathrm{v}\,\varphi^2}d\xi+C_{\delta 2}\left(\int(\mathrm{v}\psi-\ln \mathrm{v}\psi-1)d\xi+1\right),$$

$$I_5\le\delta_3\int\frac{c_\xi^2}{\mathrm{v}\,\varphi^2}d\xi+C_{\delta 3}\left(\int(\mathrm{v}\psi-\ln \mathrm{v}\psi-1)d\xi+1\right),$$

$$I_6\le\frac{1}{2}\int\frac{\theta_\xi^2}{\mathrm{v}\,\theta^2\varphi^2\gamma}d\xi+\frac{1}{2}\int\frac{c_\xi^2}{\mathrm{v}\,\varphi^2}d\xi,$$

$$I_7\le\delta_4\int\frac{c_\xi^2}{\mathrm{v}\,\varphi^2}d\xi+C_{\delta 4}\left(\int(\mathrm{v}\psi-\ln \mathrm{v}\psi-1)d\xi+1\right).$$

I_8, I_9 are assessed taking into account Lipschitzness of function $g(\rho,c,\theta)$ according to $(\varphi\theta)^{1/2}$ and inequalities

$$\frac{\left|(\varphi\theta)^{1/2}-1\right|}{\sqrt{\varphi\theta-\ln\varphi\theta-1}}\le C_9,\quad \forall (x,t)\in\Pi\,.$$

$$I_8\le N_1\int\left|(\varphi\theta)^{1/2}-1\right|\cdot\left|\mathrm{c}\gamma-1\right|d\xi\le N_1\int\frac{\left|(\varphi\theta)^{1/2}-1\right|}{\sqrt{\varphi\theta-\ln\varphi\theta-1}}\sqrt{\varphi\theta-\ln\varphi\theta-1}\cdot\left|\mathrm{c}\gamma-1\right|d\xi\le$$

$$\le N_1C_9\left(\int(\varphi\theta-\ln\varphi\theta-1)d\xi\right)^{1/2}\left(\int(c\gamma-1)^2d\xi\right)^{1/2}\le$$
$$\le N_2\left[\int\gamma(\varphi\theta-\ln\varphi\theta-1)d\xi+\frac{1}{2}\int(c\gamma-1)^2d\xi\right].$$

Further, let us break down a number axis R into domains $\Omega_i(t)$, $i=1,2$ as follows:

$\Omega_1(t)=\{x\in R:\varphi(x)\theta(x,t)\le 1\}$, $\quad\Omega_2(t)=\{x\in R:\varphi(x)\theta(x,t)>1\}$.

Then

$$I_9=\int cg\,\gamma\frac{\varphi\theta-1}{\theta}dx=\int_{\Omega_1(t)}cg\gamma\frac{\varphi\theta-1}{\theta}dx+\int_{\Omega_2(t)}cg\gamma\frac{\varphi\theta-1}{\theta}dx\le\int_{\Omega_2(t)}cg\gamma\frac{\varphi\theta-1}{\theta}dx$$

pursuant to positivity of functions $g(\rho,c,\theta)$ and $c(x,t)$.

Note that in $\Omega_2(t)$ the inequality is realized:

$$\frac{\left((\varphi\theta)^{1/2}-1\right)(\varphi\theta-1)}{\varphi\theta(\varphi\theta-\ln\varphi\theta-1)}<C_{10},\quad\forall(x,t)\in\Pi.$$

Returning to I_9, we have

$$I_9\le N_3\int_{\Omega_2(t)}\frac{\left((\varphi\theta)^{1/2}-1\right)(\varphi\theta-1)}{\varphi\theta(\varphi\theta-\ln\varphi\theta-1)}(\varphi\theta-\ln\varphi\theta-1)d\xi\le C_{10}N_3\int(\varphi\theta-\ln\varphi\theta-1)dx.$$

Fulfilling temporal integrating of the obtained from (8) inequality and using Gronuolle lemma, moving to initial variables we derive the estimate (7). Lemma is proved.

3. Auxiliary Relation between Desired Functions. Following [3], let us break down a number axis R and, accordingly, a strip Π into final segments and rectangles:

$$R=\bigcup_{N=-\infty}^{\infty}\overline{\Omega}_N,\quad\Pi=\bigcup_{N=-\infty}^{\infty}\overline{Q}_N,$$

$\Omega_N=\{x\,|\,N<x<N+1\}$, $Q_N=\Omega_N\times(0,T)$, $N=0,\pm1,\pm2,...$

Let us take at random one of such rectangles. Since in function (7) $(\mathrm{v}\psi-\ln\mathrm{v}\psi-1)$, $(\varphi\theta-\ln\varphi\theta-1)$ are negative at $\mathrm{v}>0$, $\theta>0$, then

$$U_N(t)+\int_0^t W_N(\tau)\,d\tau\le E,$$

where integrals in the definition U_N and W_N are taken according to Ω_N.

According to [3] positive constants $n(E)$, $M(E)$ exist here, which do not depend on N:

$$\frac{n(E)}{C_1}\le\int_N^{N+1}\mathrm{v}(x,t)\,dx\le M(E)C_1,\quad\frac{n(E)}{C_4}\le\int_N^{N+1}\theta(x,t)\,dx\le M(E)C_4,\ \forall t\in[0,T].\quad(11)$$

It arises from (11) that at any $t\in[0,T]$ in each domain $\overline{\Omega}_N$ there are such points $a(t)=a_N(t)\in[N,N+1]$, $a_1(t)=a_{1N}(t)\in[N,N+1]$ that

$$\frac{n(E)}{C_1}\le\mathrm{v}(a(t),t)\le M(E)C_1,\quad\frac{n(E)}{C_4}\le\theta(a_1(t),t)\le M(E)C_4.\qquad(12)$$

One auxiliary relation between desired functions in each of rectangles $\overline{Q}_N$ is derived from the first and third equations of the set (1).

$$v(x,t) = I^{-1}(t)B^{-1}(x,t)\left[v_0(x) + \int_0^t \theta(x,\tau)I(\tau)B(x,\tau)d\tau\right], \qquad (13)$$

where: $I(t) = I_N(t) = \dfrac{v_0(a(t))}{v(a(t),t)}\exp\left\{\int_0^t \theta(a(t),\tau)\,d\tau\right\}$,

$$B(x,t) = B_N(x,t) = \exp\left\{\int_{a(t)}^{x}(u_0(\xi) - u(\xi,t))d\xi\right\}.$$

True estimates are as follows:

$$0 < K_1^{-1} \le B(x,t) \le K_1, \quad 0 < K_2^{-1} \le I(t) \le K_2, \qquad \forall\, x \in \overline{\Omega}_N, \;\; \forall\, t \in [0,T]. \qquad (14)$$

The proof follows from estimates (7) and representation (13).

4. Estimates for Density (Specific Volume) and Temperature. Let $h(x,t)$ – be a continuous function. Let us introduce notations:

$$M_h(t) = \max_{|x|<\infty} h(x,t), \quad m_h(t) = \min_{|x|<\infty} h(x,t).$$

LEMMA 2. At fulfillment of the theorem conditions the following estimates are true:

$$m_v(t) \ge N_4, \quad m_\theta(t) \ge N_5, \qquad \forall\, t \in [0,T].$$

Proof. Strict positivity of specific volume follows from representation (13) taking into account theorem conditions and (7), strict positivity of temperature follows from thermal-conductivity equation of the set (1).

LEMMA 3. At fulfillment of the theorem conditions the following estimate is true:

$$M_v(t) \le N_3 \quad \forall\; t \in [0,T].$$

Proof. The following estimates take place [3]:

$$M_\theta(t) \le C_\varepsilon\, A(t) M_v(t) + C, \;\; \text{where} \;\; A(t) = \int_0^1 \frac{\theta_x^2}{v\theta^2}\,dx. \qquad (15)$$

Applying Gronuolle lemma to (13), taking into account estimates (14), (15), we shall receive boundedness of specific volume from above.

From lemma 3, taking into account (7) и (15), the following estimate arises

$$\int_0^t M_\theta(t)dt \le K_3, \qquad \forall t \in [0,T]. \qquad (16)$$

5. Estimates for Derivatives from Desired Functions. Let us multiply the second equation of the set (1) by $\dfrac{\partial}{\partial x}\left(\dfrac{1}{v}c_x\right)$ and integrate over R:

$$\frac{1}{2}\frac{d}{dt}\int \frac{1}{v}c_x^2 dx + \int\left(\frac{1}{v}c_x\right)_x^2 dx = -\frac{1}{2}\int \frac{1}{v^2}c_x^2 u_x dx + \int cg\left(\frac{1}{v}c_x\right)_x dx = I_1 + I_2. \; (17)$$

Using integration by parts, Young, Cauchy inequalities, embeddings, (9), Lipschitzness of function g according to $(\varphi\theta)^{1/2}$, we shall evaluate I_k, $k = 1,2$.

$$I_1 = -\frac{1}{2}\int \frac{1}{v^2} c_x^2 u_x dx = \int\left(\frac{1}{v}c_x\right)\left(\frac{1}{v}c_x\right)_x (u-f)dx - \frac{1}{2}\int \frac{1}{v^2} c_x^2 f'dx \le$$

$$\le \max_{x\in R}\left|\frac{1}{v}c_x\right|\left(\int\left(\frac{1}{v}c_x\right)_x^2 dx\right)^{1/2}\left(\int (u-f)^2 dx\right)^{1/2} + \frac{C_3}{2N_4}\int \frac{1}{v} c_x^2 dx .$$

As

$$\max_{x\in R}\left(\frac{1}{v}c_x\right)^2 \le 2\int\left|\frac{1}{v}c_x\left(\frac{1}{v}c_x\right)_x\right| dx \le \frac{2}{N_4^{1/2}}\left(\int \frac{1}{v}c_x^2 dx\right)^{1/2}\left(\int\left(\frac{1}{v}c_x\right)_x^2 dx\right)^{1/2},$$

then

$$I_1 \le \delta_1 \int\left(\frac{1}{v}c_x\right)_x^2 dx + C_{\delta 1}\int \frac{1}{v} c_x^2 dx .$$

Let us evaluate I_2, taking into account (9).

$$I_2 = \int cg\left(\frac{1}{v}c_x\right)_x dx \le N_1 \int\left|(\varphi\theta)^{1/2} - 1\right|\left(\frac{1}{v}c_x\right)_x dx =$$

$$= N_1 \int \frac{\left|(\varphi\theta)^{1/2} - 1\right|}{\sqrt{\varphi\theta - \ln\varphi\theta - 1}}\sqrt{\varphi\theta - \ln\varphi\theta - 1}\left(\frac{1}{v}c_x\right)_x dx \le$$

$$\le N_1 C_9 \left(\int (\varphi\theta - \ln\varphi\theta - 1)dx\right)^{1/2}\left(\int\left(\frac{1}{v}c_x\right)_x^2 dx\right)^{1/2} \le \delta_2 \int\left(\frac{1}{v}c_x\right)_x^2 dx + C_{\delta 2} .$$

Select δ_i rather small. Using integration (17) according to t, taking into account (7), we shall find:

$$\int \frac{1}{v} c_x^2 dx + \int_0^t \int\left(\frac{1}{v}c_x\right)_x^2 dx d\tau \le N_7 , \quad \forall t \in [0,T]. \tag{18}$$

Let us multiply the fourth equation of the set (1) by $(u-f)$ and integrate over R:

$$\frac{1}{2}\frac{d}{dt}\int (u-f)^2 dx + \int\left\{\frac{u_x^2}{v} + \frac{\theta}{v}f'\right\} dx = \int \frac{1}{v}u_x f'dx + \int \frac{\theta}{v}u_x dx = J_1 + J_2 . \tag{19}$$

We shall evaluate each J_k, $k = 1,2$ according to Cauchy inequality, using estimates (7), (9).

$$J_1 \le -\frac{d}{dt}\int f'(v\psi - \ln v\psi - 1)\, d\xi + C,$$

$$J_2 = \int \frac{\varphi\theta - 1}{\varphi v}u_x dx + \int \frac{1}{\varphi v}u_x dx \le -\frac{d}{dt}\int \frac{1}{\varphi}(v\psi - \ln v\psi - 1)\, d\xi + \delta\int \frac{1}{v}u_x^2 dx + C(1 + M_\theta(t))$$ Her e

$$\left|\int \frac{\varphi\theta - 1}{\varphi v}u_x dx\right| \le \frac{1}{C_4}\int \frac{\left|(\varphi\theta)^{1/2} - 1\right|\left((\varphi\theta)^{1/2} + 1\right)}{\sqrt{\varphi\theta - \ln\varphi\theta - 1}}\sqrt{\varphi\theta - \ln\varphi\theta - 1}\cdot\frac{|u_x|}{v} dx \le$$

$$\le \delta\int \frac{1}{v}u_x^2 dx + C(1 + M_\theta(t)), \quad \delta < 1 .$$

From (19), after integrating over t, taking into account (4), (7), (16), we derive:

$$\int_0^t \|u_x\|^2 d\tau \le N_8, \quad \forall t \in [0,T].$$

Arguing analogously it is possible to obtain all estimates required for proving the existence of the generalized solution. The uniqueness is proved by homogeneous equation composed regarding the difference of two possible solutions similarly to [4, 5, 6]. The existence and uniqueness of the local solution is defined in the same way as in [5, 7].

The theorem is fully proved.

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